A New Pricing Model for Valuing Employee Stock Options

by

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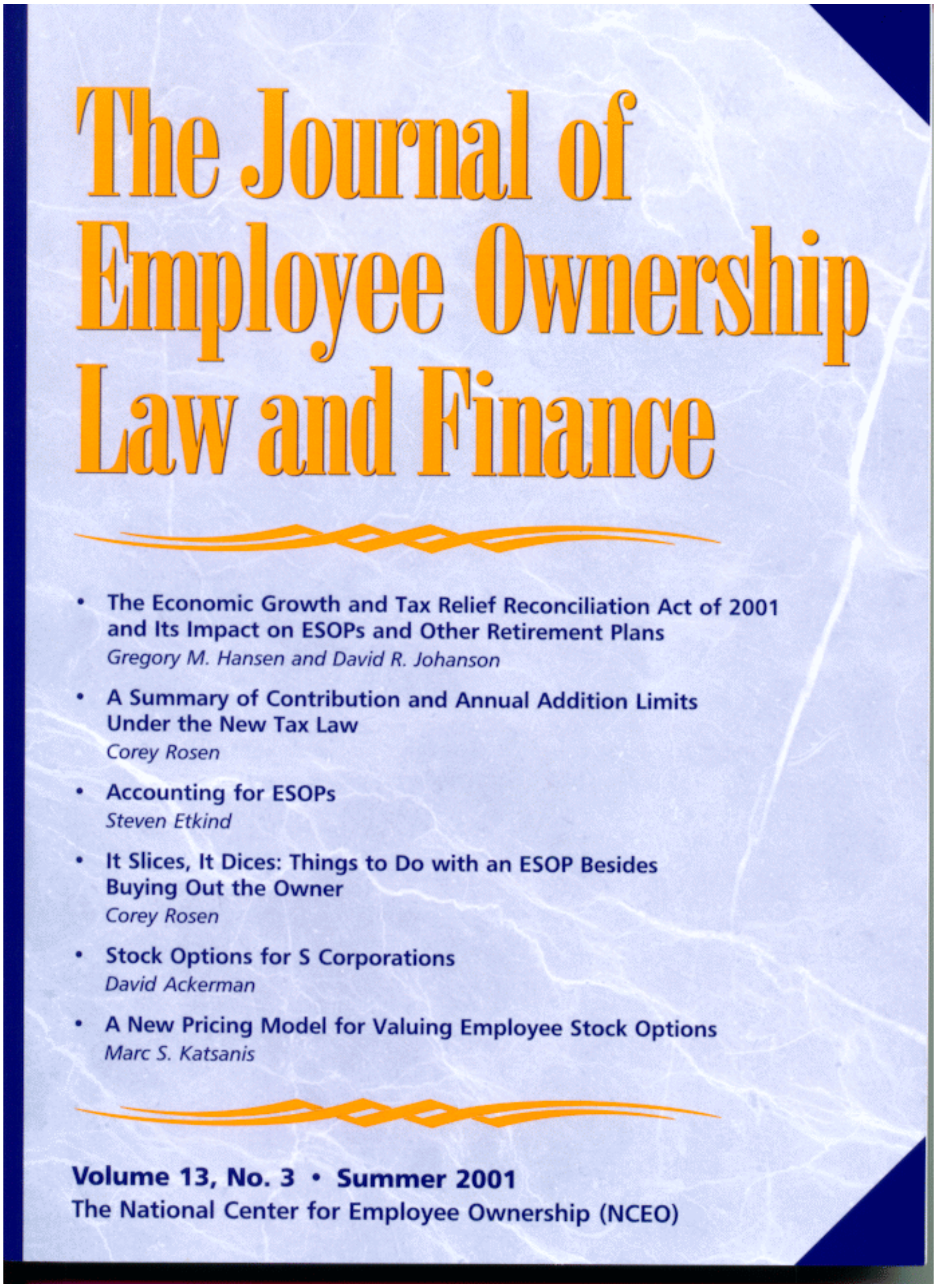

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A New Pricing Model for Valuing Employee Stock Options

Marc S. Katsanis

This article introduces the Valrex Option Pricing Model as a method to adjust a freely transferable option value (e.g. a Black-Scholes valuation) for the diminution in value related specifically to the non-transferability or lack of marketability of an employee stock option, and demonstrate how Valrex provides a more objective valuation of employee stock options than the standard Black-Scholes and binomial option pricing models.

There are several mathematical models that price freely traded options with a liquid market. However, none of these models do a good job of quantifying the discount to freely traded option prices caused by the transfer restrictions usually placed on employee stock options.

The most widely known of these freely traded models are the Black-Scholes and binomial option pricing models. The Black-Scholes model was developed by Fischer Black and Myron Scholes. The binomial model, which exists in several forms, was originally developed by John Cox, Stephen Ross and Mark Rubinstein. The trinomial option pricing model is similar to a binomial model but is more accurate at pricing complex options with characteristics similar to those found in the Valrex model. [1]

Unlike traded options, employee stock options are usually not transferable except by donation to family members. Thus, unlike a freely transferable option, where a holder has two possible means to realize liquidity, sale or exercise, a holder of an employee stock option has only one, exercise. In many cases, the fair value of an employee stock option is substantially less than an otherwise identical freely transferable option.

Background

The most widely referenced standard on valuing employee stock options is Statement 123 from the Financial Accounting Standards Board (FASB). Statement 123 requires companies, at least as a pro forma footnote, to expense the fair value of employee stock option grants over the period of time that they vest.

With respect to guidance on valuation, the FASB does not specify a single option pricing model that all companies are required to follow. It names the Black-Scholes and binomial call option pricing models as acceptable valuation methods but allows other models. Both of these valuation models are widely used in valuing traded options.

When using Black-Scholes, a binomial model or a trinomial model to value traded options, one must consider the following six input variables:

(1) Price of the underlying stock at the date of valuation
(2) Exercise price of the option
(3) Risk-free interest rate during the term of the option
(4) Expected dividend yield
(5) Expected volatility of the underlying stock
(6) Term of the option

The price of the stock (if publicly traded) can be obtained from price quotes. The exercise price of the option is set by a contractual agreement. The risk-free interest rate can be obtained from yield quotes on zero coupon government bonds. The focus of this article is on non-dividend-paying stocks, so we can assume that the expected dividend yield is 0%. The expected volatility can be estimated by calculating the historical volatility of the stock price; however, a user of the option pricing model must consider that the expected volatility may be materially different from the historical volatility and adjust the volatility input accordingly. The term of the option is set by a contractual agreement.

In Statement 123, the FASB acknowledged that the value of an option that cannot be traded is less than one that can be traded, but believed that assigning an arbitrary lack of marketability or transferability discount would be inappropriate. Instead, the FASB concluded that if holders of employee stock options desired liquidity, they would exercise their options and sell the shares. Thus, the FASB suggests substituting the expected life of the employee stock option rather than its entire term into a freely traded option pricing model such as Black-Scholes or a binomial model as a means of discounting the option value for lack of transferability. Statement 123 also states that the following factors should be considered when estimating the expected life of an employee stock option:

- *The option's vesting period.* The option's expected life should be at least equal to its vesting period. The length of time employees hold options after they first become exercisable may vary inversely with the length of the vesting period.

- *Exercise history for similar grants* (adjusted for current expectations).

- *The expected volatility of the underlying stock.* Employees tend to exercise options on high volatility stocks earlier than on those with lower volatility. Thus, because of the opportunity for early exercise, the employee stock option value varies inversely with the volatility of the underlying stock price. That is, employee stock options on highly volatile stocks are less expensive than equivalent employee stock options on stocks with low volatility (this is only the case if the employee options are in-the-money).

The expected life adjustment suggested by the FASB does, in most cases, reduce the value of a nontransferable employee stock option in relation to an otherwise identical freely traded option. However, the accuracy of the valuation is highly dependent on the option model user's subjective estimate of the expected life of the option.

In addition to the subjective nature of the expected life input, the volatility input is also widely considered to be very subjective. Not only is the determination of a volatility input subjective, but it is also material to the valuation of an option when using models such as Black-Scholes and binomial which were designed to price freely traded options. Companies acknowledge this, and many include statements along with their financial statement disclosures such as this one made in Callaway Golf Company‘s 2000 10-K:

> The Black-Scholes option valuation model was developed for use in estimating the fair value of traded options which have no vesting restrictions and are fully transferable. In addition, option valuation models require the input of highly subjective assumptions including the expected stock price volatility. Because the Company's [Callaway Golf Company's] employee stock options have characteristics significantly different from those of traded options, and because changes in subjective input assumptions can materially affect the fair value estimates, in management's opinion, the existing models do not necessarily provide a reliable single measure of the fair value of grants under the Company's employee stock-based compensation plans.

Chart I shows the fair values of various freely traded options with differing terms to expiration and volatility inputs. Chart I clearly demonstrates that changes in the time to expiration or volatility input can materially change a freely traded option's fair value.

Foundation for the Valrex Model

The fundamental concept for Valrex is based on a method developed by D.B.H. Chaffe III for quantifying the discount for lack of marketability on illiquid securities such as restricted shares (shares that have been issued without registration under the Securities Act of 1933 and are consequently restricted from being sold for a period of at least one year after issuance) of publicly traded companies by calculating the cost or price of an option to sell the shares (a put option) at the traded market price. Chaffe states further that "if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for the shares. The price of the put is the discount for lack of marketability." An application of this follows.

We will determine the discount for lack of marketability of a share of stock in a publicly traded company that cannot be sold for one year. The stock price is currently quoted at $100 per share. We will input the following assumptions into a 100 period trinomial European put option pricing model, which will result in the price of a traded put option or the discount for lack of marketability on the stock.

Inputs	
Stock Price	$100.00
Exercise Price	$100.00
Volatility	0.50
Risk Free Rate	0.05
Dividend Yield	0.00
Term to Expiration	1.00
Result	
Value of Put	16.89

The trinomial put option pricing model computes a $16.89 put option value. This $16.89 value is equivalent to what would be paid for an exchange-traded put that has a term equal to the restriction period on the restricted stock. The $16.89 value is also equal to the lack of marketability discount on the restricted stock. Thus, to find the present value of the restricted stock, subtract the put value ($16.89) from the publicly traded stock price ($100); here the result is $83.11.

Likewise, the percentage discount for lack of marketability is calculated by dividing the put value ($16.89) by the publicly traded stock price ($100); here the result is 16.89%.

Liquidity of a stock can be thought of as a form of insurance that allows one to hedge against the risk of an adverse stock price move by allowing a stockholder to sell immediately and convert the stock to cash. Purchasing a put option is an alternative means of insuring oneself against this risk.

Thus, the value or cost of the put can also be thought of as the cost to hedge most of the risk associated with the illiquidity of the restricted stock. A put option with an exercise price set at the current publicly traded price of the stock allows the holder of the stock to hedge against the risk that the price will be below $100 per share one year from now. By subtracting the value of the put option from the publicly traded value of the stock, one can strip away the portion of the stock's value that is attributable to liquidity.

Applying the Concept to Employee Stock Options

In the restricted stock example, the starting point of the valuation is the publicly-traded value of the stock. In the valuation of an employee stock option we will start with a hypothetical "freely traded" call option. This "freely traded" call option value can be computed by using a freely traded trinomial option pricing model. The next step in valuing the employee stock option is to measure and strip away the value attributable to liquidity embedded in the "freely traded" trinomial value. We can measure the embedded liquidity value by using an at-the-money put-on-call option (that is, an option to sell the employee stock option at its theoretical "freely traded" price). We can strip away the embedded liquidity value by subtracting the Valrex put-on-call value from the "freely traded" option value.[2]

Therefore, Valrex is a two-part model consisting of a "freely traded" call option pricing model and a put option on the "freely traded" call option with an exercise price set at the current value of the "freely traded" call option. The difference between the "freely traded" call option value and the put-on-call option value is the Valrex fair value of the option.

Example

To keep the example simple, we will assume that the employee stock option is fully vested and can be exercised at any time before expiration. An option that can be exercised at any time before expiration is an "American style" option. We will thus use a 100-period American style call option valuation model that will yield a value for "freely traded" option. Following are the six input assumptions for the model. (Note that the entire contractual term of the option, not the expected life, is used as an input.)

Inputs	
Stock Price	$100.00
Exercise Price	$100.00
Volatility	0.80
Risk Free Rate	0.05
Dividend Yield	0.00
Term to Expiration (Employee Option)	10.00
Result	
Value of “freely traded” call	84.08

As in the case of a restricted stock, we will determine the Valrex discount for lack of marketability from the $84.08 “freely traded” call option value by determining the value of an at-the-money put-on-call option on the employee stock option that has an exercise price equal to the $84.08 “freely traded” option value. There are eight inputs to a put-on-call option valuation model. Six of the input assumptions are identical to the input assumptions used to determine the “freely traded” value of the employee stock option shown above. The other input assumptions are the term to expiration and exercise price of the put-on-call.

The term to expiration for the put-on-call needs to be slightly less than the term to expiration of the employee stock option because we need to assume that the put-on-call expires before the employee stock option. In this case, we will make the term to expiration of the put-on-call 0.01 years less than the term of the employee stock option, so it is 9.99.

The final input assumption for a put-on-call option is its exercise price. Because we are valuing an at-the-money option (exercisable at the “freely traded” price of the employee stock option), this input is $84.08, or the “freely traded” value of the employee stock option.

These assumptions are input into the 100-period trinomial put-on-call option pricing model:

Inputs	
Stock Price	$100.00
Exercise Price (Employee Option)	$100.00
Volatility	0.80
Risk Free Rate	0.05
Dividend Yield	0.00
Term to Expiration (Employee Option)	10.00
Term to Expiration (Put-on-Call)	9.99
Exercise Price (Put-on-Call)	$84.08
Result	
Put-on-call Value	$42.85

This results in a put-on-call value of $42.85. The Valrex fair value is the difference between the "freely traded" price of the employee stock option ($84.08) and the put-on-call value ($42.85) in this example, the value is $41.23.

We can calculate the Valrex discount for lack of marketability by dividing the put-on-call value ($42.85) by the "freely traded" employee stock option price ($84.08); here the discount is 50.96%.

Chart II shows a range of Valrex discounts for lack of marketability for one- through ten-year at-the-money employee stock options on stocks with expected volatilities ranging from 20% to 150%. For the purpose of drawing this chart, the stock price and exercise price (employee option) are held constant at $100, the volatility is varied from 20% to 150%, the risk-free rate is held constant at 5%, the dividend yield is held constant at 0%, the term to expiration (employee option) is varied from one to ten years, the term to expiration (put-on-call) is varied from 0.99 years to 9.99 years (always 0.01 years less than employee option), and the exercise price (put-on-call) is equal to the "freely traded" option value using each set of assumptions.

Stock Price	$100.00
Exercise Price (Employee Option)	$100.00
Volatility	Varied from 0.20 to 1.50
Risk Free Rate	0.05
Dividend Yield	0.00
Term to Expiration (Employee Option)	Varied from 1.00 to 10.00
Term to Expiration (Put-on-Call)	Varied from 0.99 to 9.99
Exercise Price (Put-on-Call)	Calculated by using an American style "freely traded" call option model with the above assumptions

The Valrex discount for lack of marketability increases as the option goes out-of-the-money (when the stock price is below the option exercise price) and decreases as the option goes in-the-money (when the stock price exceeds the exercise price).

Another significant aspect of the Valrex model is that the value of in-the-money options (on stocks that do not pay dividends) generally decline as volatility increases.

This coincides with the observation stated by the FASB (as stated above) and others that employee stock options on highly volatile stocks are less expensive than equivalent employee stock options on stocks with low volatility. Chart III shows a range of Valrex fair values for ten-year employee stock options on stocks with expected volatilities ranging from 20% to 150%, and stock prices ranging from $10 to $200. For the purpose of drawing this chart, the stock price is varied from $10 to $200, and exercise price (employee option) is held constant at $100, the volatility is varied from 20% to 150%, the risk free rate is held constant at 5%, the dividend yield is held constant at 0%, the term to expiration (employee option) is held constant at 10 years, the term to expiration (put-on-call) is held constant at 9.99 years, and the exercise price (put-on-call) is equal to the "freely traded" option value using each set of assumptions.

Stock Price	Varied from $10.00 to $200.00
Exercise Price (Employee Option)	$100.00
Volatility	Varied from 0.20 to 1.50
Risk Free Rate	0.05
Dividend Yield	0.00
Term to Expiration (Employee Option)	10.00
Term to Expiration (Put-on-Call)	9.99
Exercise Price (Put-on-Call)	Calculated by using an American Style "freely traded" option model with the above assumptions

As shown on Chart III, going from front to back, the Valrex fair value of the option increases with volatility for out-of-the-money options, varies least in relation to volatility for at-the-money and slightly in-the-money options, and decreases as volatility increases for further in-the-money options.

For at-the-money or near-the-money employee stock options, the Valrex fair value is less sensitive to changes in the volatility assumption than option pricing models, such as Black-Scholes, that are designed to price freely traded options. Chart IV shows the Valrex fair values of options with differing terms to expiration and volatility inputs. These Valrex fair values are calculated using the same assumptions as the calculations for Chart I, except for the assumption about marketability of the option. The variation in Valrex values shown in Chart IV are less than the "freely traded" option values shown in Chart I when the volatility input changes.

Conclusion

In comparison to the Black-Scholes and binomial option valuation models mentioned in Statement 123, the Valrex model provides a more objective and reliable means of determining the fair value of an employee stock option. Valrex is more reliable and objective than Black-Scholes and binomial because it eliminates the option model user's need to make a subjective estimate about the expected life of the employee stock option, and the Valrex fair value is less sensitive to the subjective volatility input than the Black-Scholes and binomial fair values.

Bibliography

Chaffe, D.B.H. III. "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations." *Business Valuation Review.* December 1993. Pp. 182-188.

Chriss, Neil A. *Black-Scholes and Beyond.* Chicago: Irwin Professional Publishing, 1997.

Haug, Espen Gaarder. *The Complete Guide to Option Pricing Formulas.* New York: McGraw-Hill, 1998.

Hull, John C. *Options, Futures and Other Derivatives.* Third Edition. Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1997.

Mercer, Z. Christopher. *Quantifying Marketability Discounts.* Memphis, Tennessee: Peabody Publishing, LP, 1997.

Miller, Merton H. *Financial Innovations & Market Volatility*. Cambridge, Massachusetts: Blackwell Publishers, 1995.

Ravindran, K. *Customized Derivatives.* New York: McGraw-Hill, 1998.

Rodrick, Scott S., ed. *The Stock Options Book.* Third Edition. Oakland, California: The National Center for Employee Ownership, 1999.

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[1] Trinomial models are more efficient in valuing options with barrier conditions. The Valrex put-on-call has a barrier condition that prevents its value from being larger than the time value of the underlying freely transferable call value. The barrier condition prevents the Valrex value from being less than the intrinsic value of the employee stock option (assuming the option is vested). The tri- prefix in trinomial describes the structure of the decision trees used to model the future potential stock and option prices. Simply put, for every point (or node) on the decision tree there are three potential stock or option price movements until the next instant in time (or the next set of nodes). The same holds true for binomial option pricing models except that for each node on the decision tree there are only two potential future stock or option price movements.

[2] There are two basic types of options; call options and put options. A call option is the right to buy an asset (e.g. a share of stock) at a price for a period of time. Employee stock options are call options because they give employees the right to buy stock in their employer at a price for a period of time. A put option is the right to sell an asset (e.g. a share of stock or a call option in the context of the Valrex put-on-call) at a price for a period of time.

Chart I

Value of Freely-Traded Call

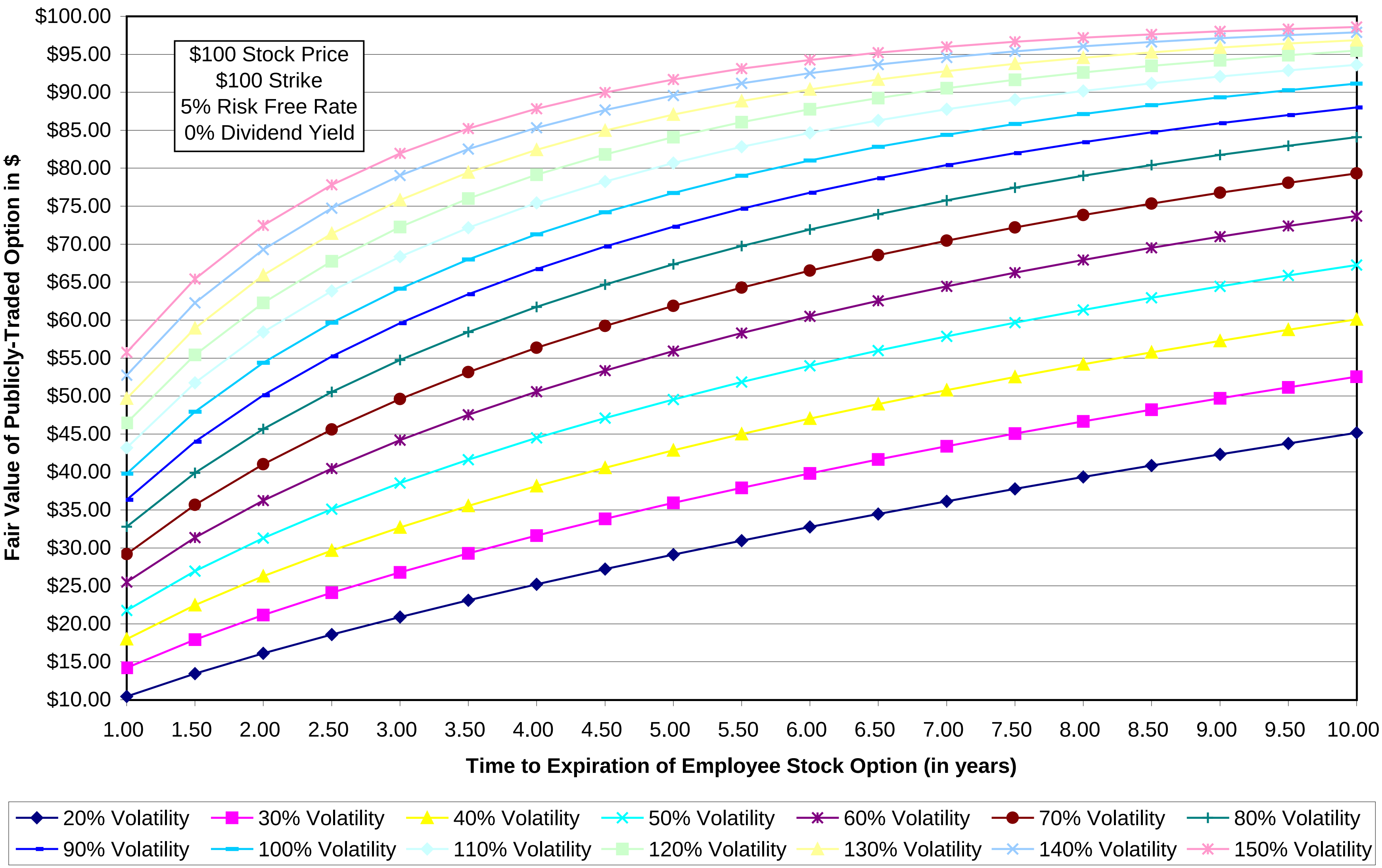


Chart II

Discount for Lack of Marketability--Valrex Discount to "freely traded" Value

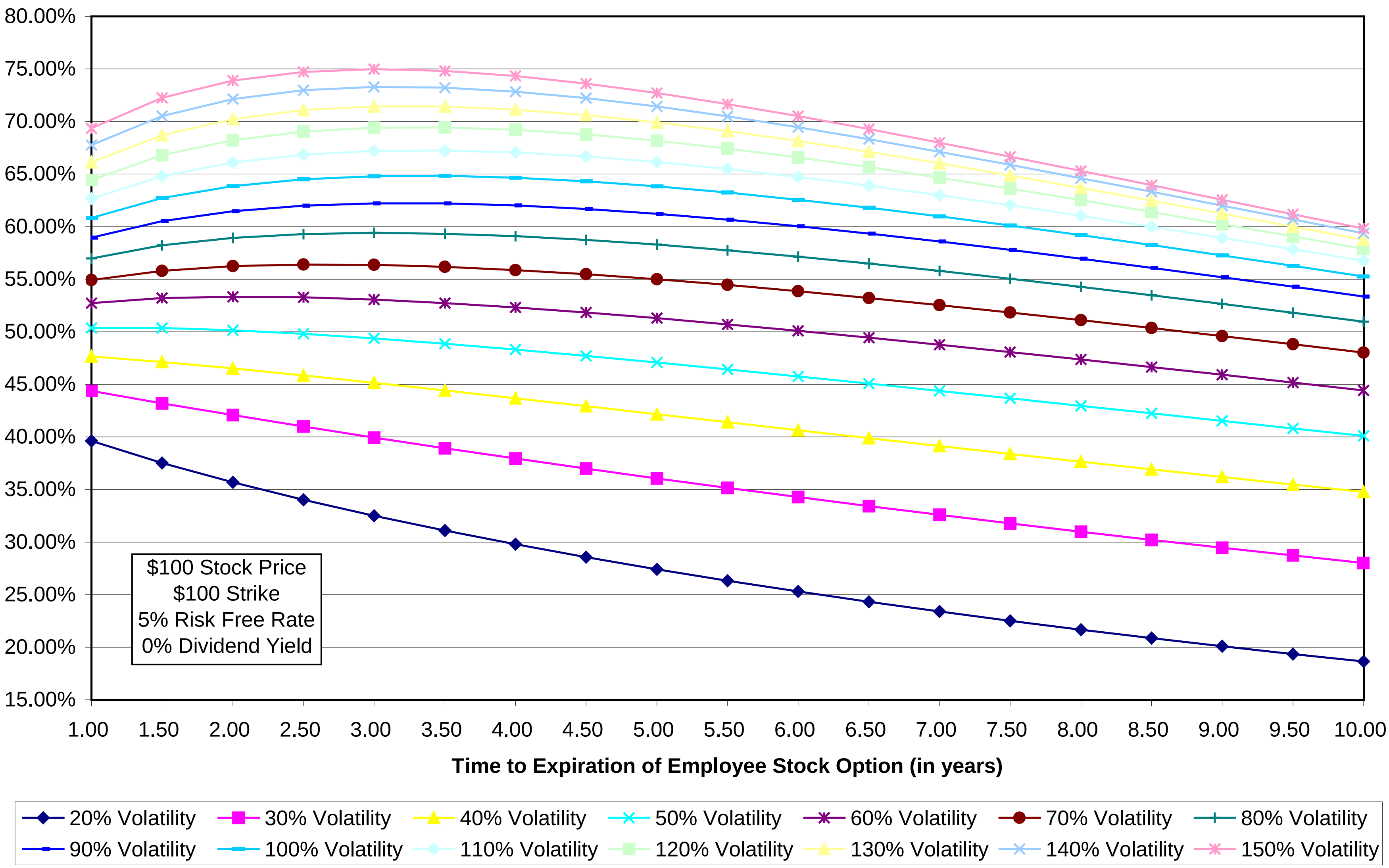
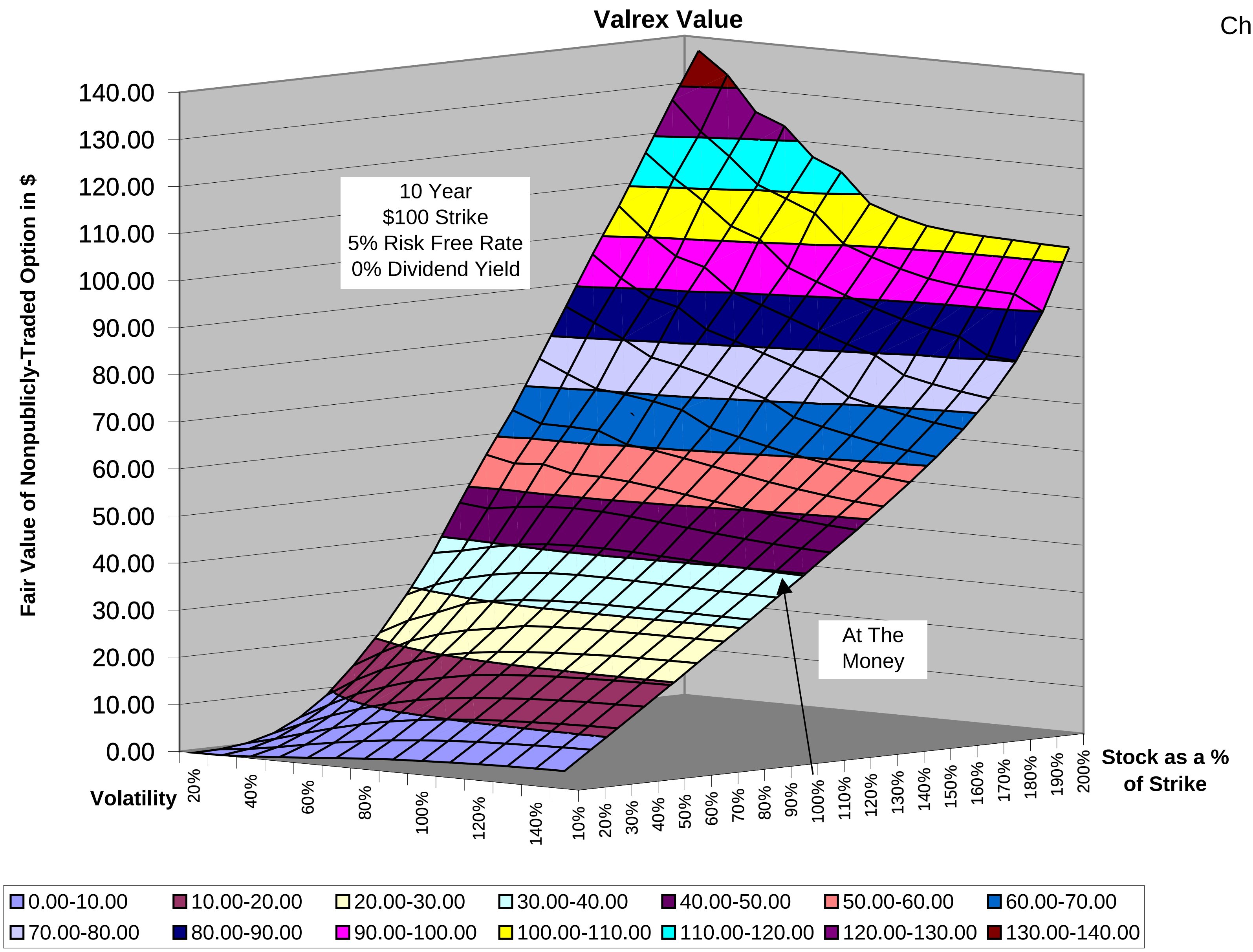


Chart III

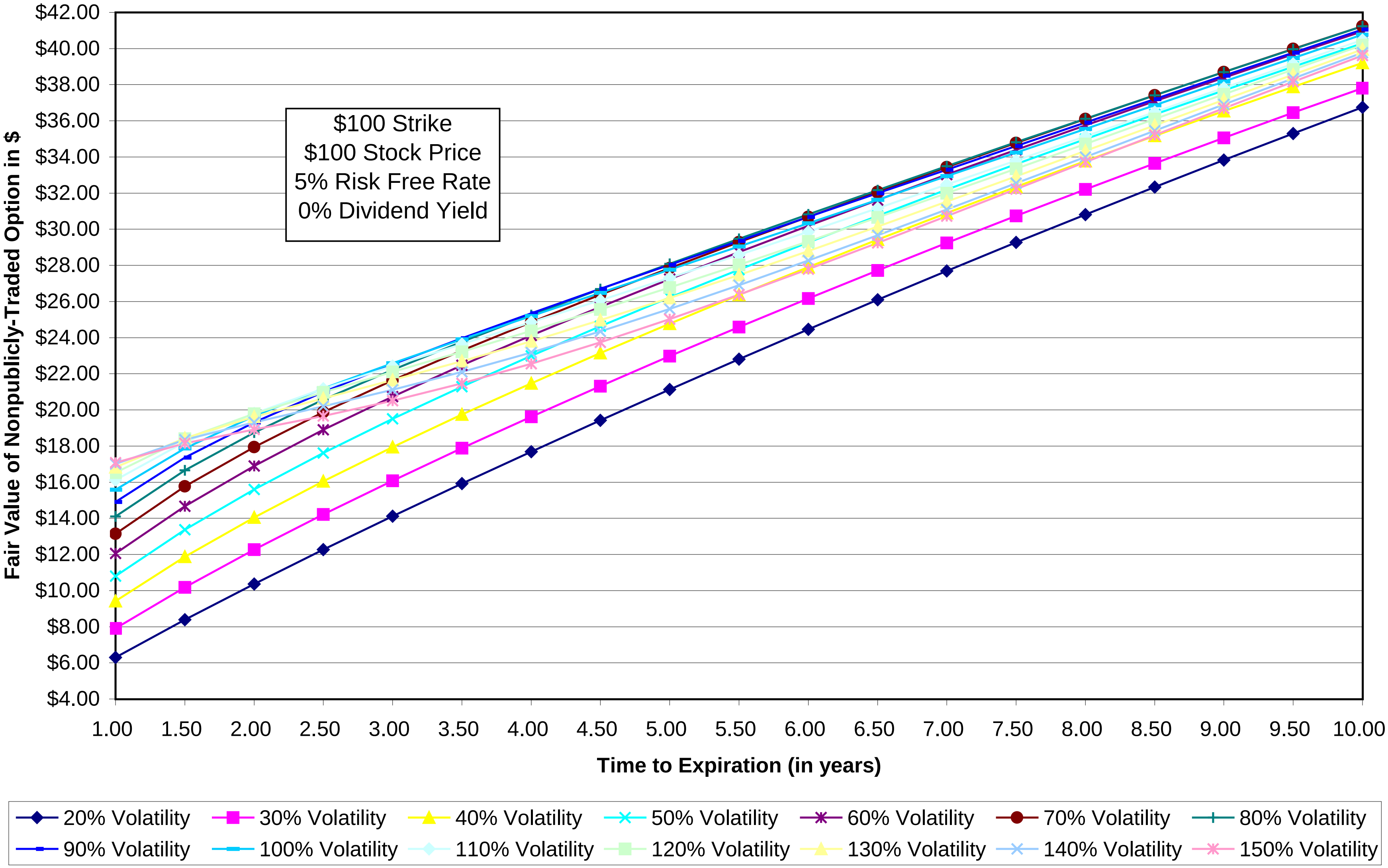


10 Year
$100 Strike
5% Risk Free Rate
0% Dividend Yield

At The
Money

Valrex Value

Chart IV



Following is a set of three trinomial tree models, which ultimately find the value of the employee stock option explained in the example. The three trinomial tree models perform the following three functions, respectively: (1) Project the forward risk neutral distribution of the stock price, (2) Find the “as if freely traded” value of the employee stock option and (3) Find the put-on-call option value. Pages 2 and 3 show the projected stock price trees and an example of how the nodes are calculated, respectively. Pages 4 and 5 show the “as if freely traded” call option tree and an example of how the nodes are calculated, respectively. Pages 6 and 7 show the put-on-call option tree and an example of how the nodes are calculated, respectively.

The trees are based on a time line that spans the term of the option. The stock price tree on page 2 starts with the first node on the left at the current stock price and jumps up, down or stays the same based on the up, middle, and down ratio multipliers calculated on page 3. The size of the multipliers is primarily based on the stock’s volatility and the time period between nodes. Applying the multipliers to fill in each node until expiration of the option completes the tree. The dispersion of the stock price returns follow a normal distribution.

Pages 4 and 5 show the result of the calculations involved in finding the “as if freely traded” call option value and examples showing how the nodes of the tree are calculated, respectively.

The “as if freely traded” call option tree is completed from right to left. Each node on the right end of the page represents the option at expiration. Thus, the value of the option is equal to its intrinsic value (stock price minus exercise price if positive and zero otherwise). Moving to the left, the values for the nodes in the earlier time periods are weighted by the up, middle and down probabilities that are calculated on page 5. Each probability weighted value is then discounted at the risk-free rate. This procedure is repeated until a value is found for the leftmost node. This is the “as if freely traded” value of the call option.

Pages 6 and 7 show the result of the calculations involved in finding the put-on-call option value and examples showing how the nodes of the tree are calculated, respectively.

The put-on-call option we are interested in is one that estimates the cost to hedge against the risk of illiquidity associated with the employee stock option.

The put-on-call option tree is completed similar to the way the “as if freely traded” call option tree is completed. The intrinsic value (“as if freely traded” option price minus exercise price if positive, zero otherwise) of an at-the-money put-on-call option is assumed at expiration, as shown in the rightmost nodes. Moving to the left, the values for the nodes in the earlier time periods are calculated as follows: (1) If the employee stock option is out-of-the-money or at-the-money at the current node, then the current node is equal to the discounted (at the risk-free rate) probability weighted (weighted by the up, middle and down probabilities) value of the three adjacent nodes in the next time period; or (2) If the employee stock option is in-the-money, then the current node is equal to the lesser of the following values: (a) the time value of the publicly-traded call or (b) the discounted (at the risk free rate) probability weighted (weighted by the up, middle and down probabilities) value of the three adjacent nodes in

The put-on-call value is then subtracted from the “as if freely traded” call value to derive the Valrex value.

Stock Price Tree

0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000
										8,193,720.982 (101)
									2,643,218.430 (82)	2,643,218.430 (102)
								852,677.762 (65)	852,677.762 (83)	852,677.762 (103)
							275,065.942 (50)	275,065.942 (66)	275,065.942 (84)	275,065.942 (104)
						88,733.723 (37)	88,733.723 (51)	88,733.723 (67)	88,733.723 (85)	88,733.723 (105)
					28,624.676 (26)	28,624.676 (38)	28,624.676 (52)	28,624.676 (68)	28,624.676 (86)	28,624.676 (106)
				9,234.055 (17)	9,234.055 (27)	9,234.055 (39)	9,234.055 (53)	9,234.055 (69)	9,234.055 (87)	9,234.055 (107)
			2,978.821 (10)	2,978.821 (18)	2,978.821 (28)	2,978.821 (40)	2,978.821 (54)	2,978.821 (70)	2,978.821 (88)	2,978.821 (108)
		960.940 (5)	960.940 (11)	960.940 (19)	960.940 (29)	960.940 (41)	960.940 (55)	960.940 (71)	960.940 (89)	960.940 (109)
	309.990 (2)	309.990 (6)	309.990 (12)	309.990 (20)	309.990 (30)	309.990 (42)	309.990 (56)	309.990 (72)	309.990 (90)	309.990 (110)
100.000 (1)	100.000 (3)	100.000 (7)	100.000 (13)	100.000 (21)	100.000 (31)	100.000 (43)	100.000 (57)	100.000 (73)	100.000 (91)	100.000 (111)
	32.259 (4)	32.259 (8)	32.259 (14)	32.259 (22)	32.259 (32)	32.259 (44)	32.259 (58)	32.259 (74)	32.259 (92)	32.259 (112)
		10.406 (9)	10.406 (15)	10.406 (23)	10.406 (33)	10.406 (45)	10.406 (59)	10.406 (75)	10.406 (93)	10.406 (113)
			3.357 (16)	3.357 (24)	3.357 (34)	3.357 (46)	3.357 (60)	3.357 (76)	3.357 (94)	3.357 (114)
				1.083 (25)	1.083 (35)	1.083 (47)	1.083 (61)	1.083 (77)	1.083 (95)	1.083 (115)
					0.349 (36)	0.349 (48)	0.349 (62)	0.349 (78)	0.349 (96)	0.349 (116)
						0.113 (49)	0.113 (63)	0.113 (79)	0.113 (97)	0.113 (117)
							0.036 (64)	0.036 (80)	0.036 (98)	0.036 (118)
								0.012 (81)	0.012 (99)	0.012 (119)
									0.004 (100)	0.004 (120)
										0.001 (121)

Time Period

Stock Price	100.000	
Strike Price	100.000	
Volatility	0.800	
Time to Expiration	10.000	
Risk Free Rate	0.050	
For. Rate/Div. Yld	**0.000**	
Up Ratio	3.0999031	(a)
Middle Ratio	1.0000000	(b)
Down Ratio	0.3225907	(c)
Up Probability	0.1470403	(d)
Middle Probability	0.4728361	(e)
Down Probability	0.3801236	(f)
Time Period Between Nodes	1.000	

Calculations for Stock Price Tree

Up, Middle and Down Ratios as shown	Corresponds to the following on Page 2	
Up Ratio = $e^{s * \sqrt{2*\Delta t}}$	(a)	Up Ratio = 3.0999031
Middle Ratio = 1	(b)	Middle Ratio = 1
Down Ratio = $e^{-s * \sqrt{2*\Delta t}}$	(c)	Down Ratio = 0.3225907

Where:

σ = Volatility	σ = 0.8
Δt = Time Between Nodes	Δt = 1

Node (1) = Current Stock Price = 100
Node (2) = Node (1) * Up Ratio = 100 * 3.09990 = 309.990
Node (3) = Node (1) * Middle Ratio = 100 * 1 = 100
Node (4) = Node (1) * Down Ratio = 100 * 0.32259 = 32.259
Node (5) = Node (2) * Up Ratio = 309.990 * 3.09990 = 960.940
Node (6) = Node (2) * Middle Ratio = 309.990 * 1 = 309.990
The remainder of the stock price tree is filled in by following the same pattern.

"As if Freely Traded" Call Option Tree

"As if Freely Traded" Value of Employee Stock Option

Time Period

0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000
										8,193,620.982 (101)
									2,643,123.308 (82)	2,643,118.430 (102)
								852,587.279 (65)	852,582.639 (83)	852,577.762 (103)
							274,979.871 (50)	274,975.458 (66)	274,970.819 (84)	274,965.942 (104)
						88,651.850 (37)	88,647.652 (51)	88,643.239 (67)	88,638.600 (85)	88,633.723 (105)
					28,546.796 (26)	28,542.803 (38)	28,538.606 (52)	28,534.193 (68)	28,529.553 (86)	28,524.676 (106)
				9,161.304 (17)	9,156.594 (27)	9,152.182 (39)	9,147.984 (53)	9,143.572 (69)	9,138.932 (87)	9,134.055 (107)
			2,916.454 (10)	2,910.314 (18)	2,904.098 (28)	2,898.105 (40)	2,892.750 (54)	2,888.337 (70)	2,883.698 (88)	2,878.821 (108)
		914.549 (5)	908.537 (11)	901.816 (19)	894.390 (29)	886.360 (41)	878.072 (55)	870.456 (71)	865.817 (89)	860.940 (109)
	280.346 (2)	276.031 (6)	271.000 (12)	265.107 (20)	258.164 (30)	249.931 (42)	240.106 (56)	228.363 (72)	214.867 (90)	209.990 (110)
83.474 (1)	81.045 (3)	78.202 (7)	74.852 (13)	70.869 (21)	66.080 (31)	60.226 (43)	52.885 (57)	43.264 (73)	29.371 (91)	0.000 (111)
	21.599 (4)	20.088 (8)	18.339 (14)	16.307 (22)	13.935 (32)	11.158 (44)	7.899 (58)	4.108 (74)	0.000 (92)	0.000 (112)
		4.495 (9)	3.790 (15)	3.022 (23)	2.203 (33)	1.363 (45)	0.575 (59)	0.000 (75)	0.000 (93)	0.000 (113)
			0.622 (16)	0.414 (24)	0.227 (34)	0.080 (46)	0.000 (60)	0.000 (76)	0.000 (94)	0.000 (114)
				0.037 (25)	0.011 (35)	0.000 (47)	0.000 (61)	0.000 (77)	0.000 (95)	0.000 (115)
					0.000 (36)	0.000 (48)	0.000 (62)	0.000 (78)	0.000 (96)	0.000 (116)
						0.000 (49)	0.000 (63)	0.000 (79)	0.000 (97)	0.000 (117)
							0.000 (64)	0.000 (80)	0.000 (98)	0.000 (118)
								0.000 (81)	0.000 (99)	0.000 (119)
									0.000 (100)	0.000 (120)
										0.000 (121)

Stock Price	100.000	
Strike Price	100.000	
Volatility	0.800	
Time to Expiration	10.000	
Risk Free Rate	0.050	
For. Rate/Div. Yld	0.000	
Up Ratio	3.0999031	(a)
Middle Ratio	1.0000000	(b)
Down Ratio	0.3225907	(c)
Up Probability	0.1470403	(d)
Middle Probability	0.4728361	(e)
Down Probability	0.3801236	(f)
Time Period Between Nodes	1.000	

Calculations for "As if Freely Traded" Call Option Tree

Up, Middle and Down Probabilities as shown

Corresponds to the following on Page 4

$$p_u = \left(\frac{e^{b\Delta t/2} - e^{-s\sqrt{\Delta t/2}}}{e^{s\sqrt{\Delta t/2}} - e^{-s\sqrt{\Delta t/2}}} \right)^2$$

(d) $p_u = 0.1470403$

$p_m = 1 - p_u - p_d$

(e) $p_m = 0.4728361$

$$p_d = \left(\frac{e^{s\sqrt{\Delta t/2}} - e^{b\Delta t/2}}{e^{s\sqrt{\Delta t/2}} - e^{-s\sqrt{\Delta t/2}}} \right)^2$$

(f) $p_d = 0.3801236$

Where

p_u = Up Probability
p_m = Middle Probability
p_d = Down Probability

σ = Volatility	σ = 0.8
b = Risk Free Rate - Dividend Yield	b = 0.05 - 0.00 = 0.05
Δt = Time Between Nodes	Δt = 1 year

Starting from the Top right hand corner of the tree

Node (101) = Max(Intrinsic value of option or 0)= 8,193,720.982 - 100 = 8,193,620.982
Node (102) = Max(Intrinsic value of option or 0)= 2,643,218.430 - 100 = 2,643,118.430
Node (103) = Max(Intrinsic value of option or 0)= 852,677.762 - 100 = 852,577.762

Going backward from period 10 to period 9, apply a probability and a discount factor

Thus
Node (82) = $e^{-r\Delta t}$ * (p_u * Node (101) + p_m * Node (102) + p_d * Node (103)) or
Node (82) = $e^{(-0.05*1)}$ * (0.1470403 * 8,193,620.982 + 0.4728361 * 2,643,118.430 + 0.3801236 * 852,577.762)
Node (82) =2,643,123

The remaining node values are calculated by probability weighting and discounting back the three nodes to the right.

Following this pattern will ultimately solve for the option value at Node (1)

The value for this option is 83.47, which is the publicly-traded value of the employee stock option and the exercise price of the put-on-call option.

Put-on-Call Option Tree

Discount for Lack of Marketability of Employee Stock Option

Time Period

0.000	0.999	1.998	2.997	3.996	4.995	5.994	6.993	7.992	8.991	9.990
										0.000 (101)
									0.000 (82)	0.000 (102)
								0.000 (65)	0.000 (83)	0.000 (103)
							0.000 (50)	0.000 (66)	0.000 (84)	0.000 (104)
						0.000 (37)	0.000 (51)	0.000 (67)	0.000 (85)	0.000 (105)
					0.083 (26)	0.000 (38)	0.000 (52)	0.000 (68)	0.000 (86)	0.000 (106)
				2.910 (17)	1.179 (27)	0.230 (39)	0.000 (53)	0.000 (69)	0.000 (87)	0.000 (107)
			13.360 (10)	10.153 (18)	6.548 (28)	2.975 (40)	0.637 (54)	0.000 (70)	0.000 (88)	0.000 (108)
		27.619 (5)	25.817 (11)	23.194 (19)	19.480 (29)	14.322 (41)	7.436 (55)	1.763 (71)	0.000 (89)	0.000 (109)
	38.697 (2)	39.075 (6)	39.102 (12)	38.623 (20)	37.382 (30)	34.910 (42)	30.115 (56)	18.373 (72)	4.877 (90)	0.000 (110)
44.289 (1)	46.007 (3)	47.728 (7)	49.439 (13)	51.125 (21)	52.780 (31)	54.420 (43)	56.214 (57)	59.856 (73)	67.727 (91)	83.474 (111)
	50.285 (4)	52.748 (8)	55.369 (14)	58.188 (22)	61.276 (32)	64.767 (44)	68.928 (58)	73.903 (74)	79.406 (92)	83.474 (112)
		54.985 (9)	57.877 (15)	60.967 (23)	64.281 (33)	67.843 (45)	71.629 (59)	75.538 (75)	79.406 (93)	83.474 (113)
			58.649 (16)	61.713 (24)	64.939 (34)	68.324 (46)	71.857 (60)	75.538 (76)	79.406 (94)	83.474 (114)
				61.843 (25)	65.021 (35)	68.356 (47)	71.857 (61)	75.538 (77)	79.406 (95)	83.474 (115)
					65.026 (36)	68.356 (48)	71.857 (62)	75.538 (78)	79.406 (96)	83.474 (116)
						68.356 (49)	71.857 (63)	75.538 (79)	79.406 (97)	83.474 (117)
							71.857 (64)	75.538 (80)	79.406 (98)	83.474 (118)
								75.538 (81)	79.406 (99)	83.474 (119)
									79.406 (100)	83.474 (120)
										83.474 (121)

Stock Price	100.000	
Strike--Opt. On Stock	100.000	
Strike--Opt. on Opt.	83.474	
Volatility	0.800	
Time to Expiration	9.990	
Risk Free Rate	0.050	
For. Rate/Div. Yld	0.000	
Up Ratio	3.0999031	(a)
Middle Ratio	1.0000000	(b)
Down Ratio	0.3225907	(c)
Up Probability	0.1470829	(d)
Middle Probability	0.4728620	(e)
Down Probability	0.3800551	(f)
Time Period Between Nodes	0.999	

Calculations for Put-on-Call Option Tree

Up, Middle and Down Probabilities as shown

Corresponds to the following on Page 6

$$p_u = \left(\frac{e^{b\Delta t/2} - e^{-s\sqrt{\Delta t/2}}}{e^{s\sqrt{\Delta t/2}} - e^{-s\sqrt{\Delta t/2}}} \right)^2$$

(d) $p_u = 0.1470829$

$$p_m = 1 - p_u - p_d$$

(e) $p_m = 0.4728620$

$$p_d = \left(\frac{e^{s\sqrt{\Delta t/2}} - e^{b\Delta t/2}}{e^{s\sqrt{\Delta t/2}} - e^{-s\sqrt{\Delta t/2}}} \right)^2$$

(f) $p_d = 0.3800551$

Where

p_u = Up Probability
p_m = Middle Probability
p_d = Down Probability

σ = Volatility	σ = 0.8
b = Risk Free Rate - Dividend Yield	b = 0.05 - 0.00 = 0.05
Δt = Time Between Nodes	Δt = 0.999 years

Starting from the bottom right hand corner of the tree

Node (119) = Intrinsic value of the put-on-call [Max(Strike Price of Compound - Value of Call Option or 0)] = 83.474 - 0 = 83.474
Node (120) = Intrinsic value of the put-on-call [Max(Strike Price of Compound - Value of Call Option or 0)] = 83.474 - 0 = 83.474
Node (121) = Intrinsic value of the put-on-call [Max(Strike Price of Compound - Value of Call Option or 0)] = 83.474 - 0 = 83.474

Going backward from period 9.99 to period 8.991, we must consider whether the put-on-call is in-the-money or out-of-the-money. The call is out-of-the-money at node (100) because the stock price is $0.004 and the exercise price is $100.

Thus
Node (100) = $e^{-r\Delta t}$ * (p_u * Node (119) + p_m * Node (120) + p_d * Node (121)) or
Node (100) = $e^{(-0.05 * 0.999)}$ * (0.1470829 * 83.474 + 0.4728620 * 83.474 + 0.3800551 * 83.474)
Node (100) = 79.406

Here is an example of how a node is calculated when the employee stock option is in-the-money. We will calculate node (41). Node (41) represents a point where the employee stock option is in-the-money because stock price is $961.94 and the exercise price is $100.

Thus

Node (41) = Min [$e^{-r\Delta t}$ * (p_u * Node (54) + p_m * Node (55) + p_d * Node (56)) or time value of the "as if" traded stock option]
Node (41) = Min [$e^{(-0.05 * 0.999)}$ * (0.1470829 * 0.637 + .4728620 * 7.436 + 0.3800551 * 30.115) or 24.42]
Node (41) = Min [14.32 or 24.42]
Node (41) = 14.32

Following this pattern will ultimately solve for the option value at Node (1)

The value of the put-on-call option is 44.289.

This is 53.06% of the value of the underlying call which is an indication of the discount for lack of marketability of the employee stock option.